Multimode Network Description of a Planar Periodic Metal-Strip Grating at a Dielectric Interface—Part I: Rigorous Network Formulations

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Abstract —In Part I of this set of two papers, the problem of a plane wave incident at an arbitrary angle on a metal-strip grating of arbitrary period located at an air–dielectric interface is formulated rigorously in terms of a pair of "static" integral equations, from which new equivalent *multimode* network descriptions are derived. Both aperture and obstacle approaches are treated, and both TE and TM polarizations are considered explicitly. In Part II we present two approximate, but very simple and accurate, analytical solutions of the relevant integral equations, which in turn lead to simple and useful multimode network descriptions of the discontinuity. Several numerical comparisons are also presented between the results obtained using these new simple networks and those from an independent numerical reference solution. Excellent agreement is found over a fairly large range of parameter values.

I. Introduction

ONE OF THE problems in electromagnetic scattering that has always attracted much attention is the scattering of waves by various types of transmission gratings. Starting with the work of Lord Rayleigh [1] and continuing up to the present time, continual efforts have been made to improve the understanding of this phenomenon. Within this context, the objective of this paper is to present a set of new multimode equivalent network representations for the scattering by a transmission grating composed of a zero-thickness, periodic metal-strip grating at a plane dielectric interface of the type shown in Fig. 1.

For this scattering problem, rigorous solutions are available only for a self-reciprocal and symmetric structure [2]–[4] ($a/p = 1/2$ and $\epsilon_r^{(1)} = \epsilon_r^{(2)}$ in Fig. 1). Other asymptotically rigorous or approximate analyses [5]–[15] can also be found, and some of the solutions do provide very accurate numerical results. The equivalent network representations that are available in the literature are far more limited, however. When the grating is operated under single-mode conditions, i.e., there is present only one reflected wave and only one transmitted wave, the solution for the equivalent network in the *Waveguide Handbook* [16] is valid for arbitrary incidence angle but only when the same medium is present on both sides of the grating. When the grating is placed at an air–dielectric interface, equivalent network results are presented [17], [18] for the special case of normal incidence only, and under single-mode conditions. In these two papers [17], [18], comparisons are made with other solutions, and some errors in the literature are identified. An equivalent network is also available [19] in a restricted multimode case, when only one higher mode is propagating, meaning that only a single diffracted wave (or higher spectral order) is present on each side in addition to the basic reflected and transmitted waves. That solution is valid, furthermore, only when the same medium is present on both sides; in addition, it is given for TM incidence only, and only for the small-aperture range (small spacing between grating strips).

In the present pair of papers we first present new equivalent network representations of the metal-strip grating that can accommodate at the same time a different medium on each side of the grating, arbitrary values of the

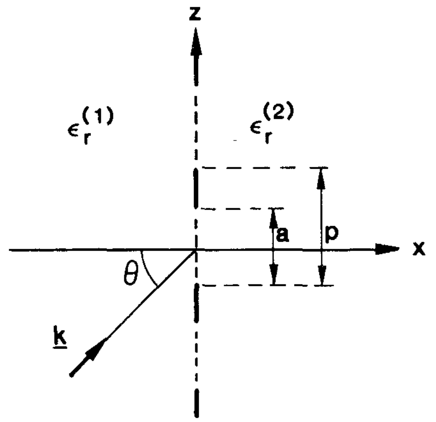

Fig. 1. The structure under examination is a zero-thickness metal-strip grating placed at a plane air–dielectric interface. The excitation is a plane wave incident at an angle; both TE and TM polarizations are considered. The ratio between the period p and the wavelength λ_0 of the incident plane wave is such that higher propagating modes (or spectral orders) must be considered.

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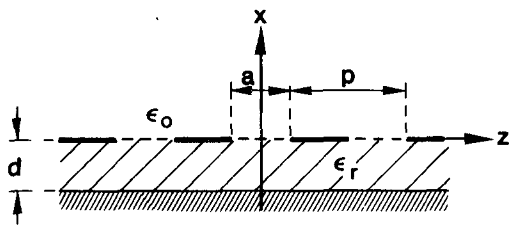

Fig. 2. The original motivation for this work is the study of a class of leaky-wave antennas for which the discontinuity in Fig. 1 is a key constituent. (The grating is here rotated through 90°.)

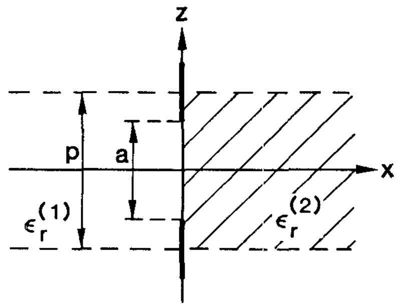

Fig. 3. Following the unit cell viewpoint, the structure in Fig. 1 can be decomposed into an infinite number of "aperture" unit cells.

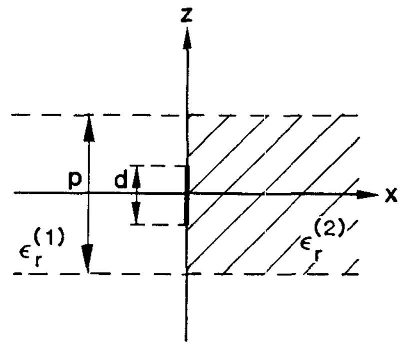

Fig. 4. An alternative decomposition is the "obstacle" unit cell, obtained by placing the origin of the coordinate system at the center of an obstacle rather than an aperture.

relative aperture size, arbitrary angles of incidence, and an arbitrary number of higher modes. In this context, we derive integral equations and new rigorous multimode network formulations that satisfy all these requirements. In Part II, we derive explicit small-argument solutions to these integral equations which are in a particularly simple analytical form, and we also obtain correspondingly simplified equivalent networks.

The principal advantage associated with the use of equivalent network representations is that such a description of the grating can then be employed as a constituent part of a variety of more complex structures, thereby permitting greater insight into the performance properties and simplifying the eventual design. Our original motivation was the study of a particular class of leaky-wave antennas (see Fig. 2) in which the grating on an air–dielectric interface is seen to serve as a key structural constituent.

In the following sections we present solutions that are analytical and in network form which are obtained by analyzing the discontinuity in Fig. 1. The presentation has been divided, for the sake of clarity, into two parts. Part I (this paper) is concerned with the mathematical formulation of the problem for aperture and obstacle phrasings, and for both TE and TM excitations, together with the new formal multimode network representations of the solutions. In Part II we derive small-obstacle and small-aperture solutions for the relevant integral equations that provide very simple and accurate network descriptions of the discontinuity. In the final section of Part II, we also present several numerical comparisons between the results obtained by using the networks developed and those from an independent numerical reference solution, showing the accuracy and flexibility of these equivalent network descriptions.

In Section II of this paper, we begin by laying some foundations relating to our unit cell approach and to the use of static characteristic impedances, which permit us to employ simplified kernels in the resulting rigorous integral equations and to obtain useful equivalent network representations for the discontinuity. Two alternative approaches, an aperture approach and an obstacle approach, are possible in the derivation of integral equations that characterize the discontinuity, and the formulations corresponding to these approaches are presented in Sections III and IV, respectively. It is to be noted that the equivalent networks developed from these formulations are somewhat different from each other. Application to either TE or TM excitation follows directly upon the insertion of the appropriate expression for the modal characteristic impedance. Explicit expressions for each of the four cases, combining aperture or obstacle formulations with TE or TM excitation, are presented in Section V. It is also shown there that of these four cases only two different types of integral equation result; they are both Fredholm integral equations of the first kind with singular kernels, but the kernels in each are different. In Part II, explicit analytical "small-argument" solutions are derived for each of these two integral equations; they lead to very simple and accurate network descriptions that are valid over a reasonably wide range of parameter values.

II. Background: The Unit Cell Approach and the Static Impedance

The first step toward the solution of the scattering problem posed by a plane wave incident from the left on the structure in Fig. 1 is the recognition that due to its periodicity and infinite extent the grating can be decomposed into an infinite sequence of identical *unit cells*. Two such unit cells are of particular interest: one obtained by placing the origin of the coordinate system at the center of an aperture and the other at the center of an obstacle, as in Figs. 3 and 4.

Following this viewpoint, we effectively reduce the open scattering problem to a waveguide problem. In fact, the unit cell itself is equivalent to a waveguide where the walls are phase-shift walls whose properties depend on the angle of incidence of the incident plane wave. The advantages of this viewpoint are that we can now use a modal decomposition of the fields and that the results obtained from the analysis will be naturally compatible with a network description of the phenomenon. A detailed derivation of the vector mode functions $\boldsymbol{e}_n(z)$ and $\boldsymbol{h}_n(z)$ can be found in [19], but a short summary is included here in the Appendix for clarity.

Now that we have available a complete set of mode functions, we can decompose the total transverse field components in the form

$$\boldsymbol{E}_t(x,z)=\sum_{n=-\infty}^{+\infty} V_n(x)\boldsymbol{e}_n(z) \tag{1}$$

$$\boldsymbol{H}_t(x,z)=\sum_{n=-\infty}^{+\infty} I_n(x)\boldsymbol{h}_n(z). \tag{2}$$

In addition to the modal decompositions in (1) and (2), another important concept that we employ later is that of *static characteristic impedance*. Depending on which polarization we employ, from now on we will use the following definitions: for TM modes,

$$\frac{1}{Z_{n,s}^{(m)}}=Y_{n,s}^{(m)}=\frac{\omega\epsilon_0\epsilon_r^{(m)}}{\lim_{\omega\to 0}k_{x,n}^{(m)}}=+j\frac{\omega\epsilon_0\epsilon_r^{(m)}p}{2\pi|n|},\qquad n\neq 0 \tag{3}$$

and for TE modes,

$$Z_{n,s}^{(m)}=\frac{1}{Y_{n,s}^{(m)}}=\frac{\omega\mu_0\mu_r^{(m)}}{\lim_{\omega\to 0}k_{x,n}^{(m)}}=+j\frac{\omega\mu_0\mu_r^{(m)}p}{2\pi|n|},\qquad n\neq 0 \tag{4}$$

where the subscript s stands for "static" and the positive sign is a direct consequence of the time dependence $\exp[j\omega t]$ chosen and suppressed. The superscript (m) signifies either region (1) or (2) in Fig. 1, and in (3) and (4) we recognize that

$$k_{x,n}^{(m)}=\sqrt{k_0^2\epsilon_r^{(m)}\mu_r^{(m)}-k_{z,n}^2},\qquad k_{z,n}=k_{z,0}+2n\pi/p$$

consistent with (A5) and (A11) of the Appendix.

As a consequence of the above definitions, we are now ready to introduce two separate formulations: one for the aperture approach and another for the obstacle approach. The distinction between TE and TM excitation is made later via the substitution of the proper expression for the static characteristic impedance or admittance.

III. Aperture Formulation

Using the field decomposition in (2) and imposing the continuity of the total transverse magnetic field at the aperture in Fig. 3, we have

$$\sum_{n=-\infty}^{+\infty} I_n^{(1)}(0)\boldsymbol{h}_n(z)-\sum_{n=-\infty}^{+\infty} I_n^{(2)}(0)\boldsymbol{h}_n(z)=0 \tag{5}$$

where $I_n^{(m)}$ is the total modal current, and the superscripts (1) and (2) refer to $x\leq 0$ and $x\geq 0$, respectively. Note that since (5) represents the continuity of the total transverse magnetic field, it also contains the exciting plane wave, as a part of the $n=0$ component of the first summation. Therefore, no explicit reference needs to be made with respect to the excitation.

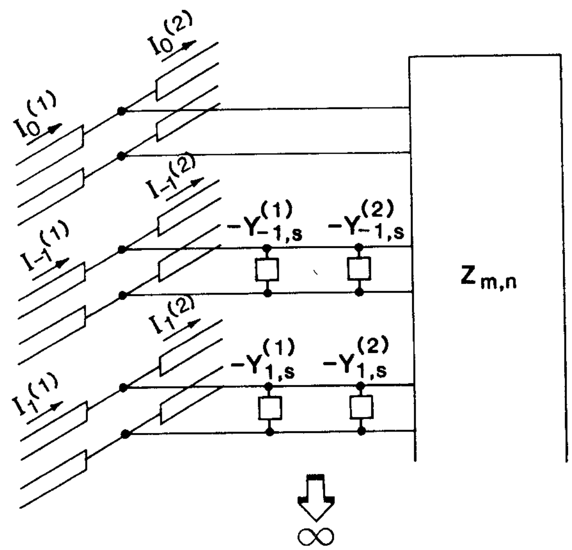

Fig. 5. Following the aperture formulation, the solution to the scattering problem posed by the structure in Fig. 1 can be phrased in terms of an equivalent shunt network coupling all of the higher modes excited by the grating.

From (5) we will now develop a rigorous integral equation whose kernel consists of a sum of *static*, rather than dynamic, modes. This procedure facilitates the solution of the integral equation and also leads to a very simple and useful network formulation.

Following the development in the *Waveguide Handbook* [20], a key step toward the formulation of such a "static" integral equation is to add the terms

$$\sum_{n\neq 0} V_n\left(Y_{n,s}^{(1)}+Y_{n,s}^{(2)}\right)\boldsymbol{h}_n(z) \tag{6}$$

to both sides of (5), obtaining

$$\sum_{n=-\infty}^{+\infty}\left(I_n^{(1)}-I_n^{(2)}\right)\boldsymbol{h}_n(z)+\sum_{n\neq 0} V_n\left(Y_{n,s}^{(1)}+Y_{n,s}^{(2)}\right)\boldsymbol{h}_n(z) = \sum_{n\neq 0} V_n\left(Y_{n,s}^{(1)}+Y_{n,s}^{(2)}\right)\boldsymbol{h}_n(z) \tag{7}$$

where the static characteristic admittance terms are defined in (3) and (4). The left-hand side of (7) suggests the very useful current redefinition

$$\overline{I}_n=\begin{cases}\left(I_0^{(1)}-I_0^{(2)}\right) & \text{for } n=0\\ \left(I_n^{(1)}-I_n^{(2)}\right)+V_n\left(Y_{n,s}^{(1)}+Y_{n,s}^{(2)}\right) & \text{for } n\neq 0\end{cases} \tag{8}$$

which leads to the *equivalent network* interpretation shown in Fig. 5, where the modal voltages V_m are given by

$$V_m=\sum_{n=-\infty}^{+\infty}\overline{I}_nZ_{m,n}. \tag{9}$$

The above analytical expressions are equivalent to those developed in the *Waveguide Handbook* [20], but the network representation for them presented in Fig. 5 is new, and, as we shall see, is very useful and practical in its reduced form given in Part II.

It is now appropriate to recall the transform relation between the voltage V_m of the mth mode and the electric field $\boldsymbol{E}$ in the aperture, namely,

$$V_m = \int_{\text{ap}} (\boldsymbol{x}_0 \times \boldsymbol{E}) \cdot \boldsymbol{h}_m^*(z')\,dz' \tag{10}$$

where the star means complex conjugate. Next, we substitute (10) into (7) taking (8) into account and obtaining, after a simple manipulation,

$$\sum_{n=-\infty}^{+\infty} \bar{I}_n \boldsymbol{h}_n(z) = \int_{\text{ap}} (\boldsymbol{x}_0 \times \boldsymbol{E}) \cdot \sum_{m \neq 0} \left(Y_{m,s}^{(1)} + Y_{m,s}^{(2)}\right) \boldsymbol{h}_m(z) \boldsymbol{h}_m^*(z')\,dz'. \tag{11}$$

Equation (11) is the fundamental integral equation we are seeking; note that the kernel is simplified, because it consists of a sum of static terms, but that the "incident" excitation is now more complicated.

Due to the linearity of the problem, however, a more useful equation can be obtained by expanding the unknown aperture magnetic current $\boldsymbol{x}_0 \times \boldsymbol{E}(z')$ into a sum of partial magnetic currents $M_n(z')$, defined by

$$(\boldsymbol{x}_0 \times \boldsymbol{E}(z')) = \boldsymbol{A}_0(z') \sum_{n=-\infty}^{+\infty} \bar{I}_n M_n(z') \tag{12}$$

where $\boldsymbol{A}_0(z')$ is an appropriate vector function to be defined later. Substituting (12) into (11) and comparing like coefficients of $\bar{I}_n$, we finally obtain the following set of partial integral equations:

$$\boldsymbol{h}_n(z) = \int_{\text{ap}} M_n(z') \boldsymbol{A}_0(z') \cdot \sum_{m \neq 0} \left(Y_{m,s}^{(1)} + Y_{m,s}^{(2)}\right) \boldsymbol{h}_m(z) \boldsymbol{h}_m^*(z')\,dz' \tag{13}$$

where $n = 0, \pm 1, \pm 2, \cdots$.

It is important to note that, although (13) has a kernel composed of static terms, it indeed represents a *rigorous* formulation of the problem under investigation.

To conclude the network phrasing of the problem, let us now substitute (12) into (10) and obtain, after inverting the order of summation and integration,

$$V_m = \sum_{n=-\infty}^{+\infty} \bar{I}_n \int_{\text{ap}} M_n(z') \boldsymbol{A}_0(z') \cdot \boldsymbol{h}_m^*(z')\,dz'. \tag{14}$$

Comparing (14) with (9), we finally obtain

$$Z_{m,n} = \int_{\text{ap}} M_n(z') \boldsymbol{A}_0(z') \cdot \boldsymbol{h}_m^*(z')\,dz' \tag{15}$$

where $Z_{m,n}$ is the generic element of the impedance matrix description introduced earlier (see Fig. 5 and (9)). Equation (15), together with (13) and the network in Fig. 5, constitutes the formal solution of the aperture phrasing of the scattering problem under investigation.

IV. Obstacle Formulation

To phrase the problem in terms of the obstacle rather than the aperture, we start by imposing the condition that the tangential electric field components vanish on both sides of the obstacle in Fig. 4. We thus obtain

$$\boldsymbol{E}_t^{(1)} = 0 = \sum_{n=-\infty}^{+\infty} V_n^{(1)} \boldsymbol{e}_n(z) \qquad \text{for } x = 0^- \tag{16}$$

$$\boldsymbol{E}_t^{(2)} = 0 = \sum_{n=-\infty}^{+\infty} V_n^{(2)} \boldsymbol{e}_n(z) \qquad \text{for } x = 0^+. \tag{17}$$

Then, using the static characteristic impedance previously defined in (3) and (4), we add to both sides of (16) and (17), respectively, the terms

$$\sum_{n \neq 0} Z_{n,s}^{(1)} I_n^{(1)} \boldsymbol{e}_n(z) \tag{18}$$

$$-\sum_{n \neq 0} Z_{n,s}^{(2)} I_n^{(2)} \boldsymbol{e}_n(z) \tag{19}$$

obtaining

$$\sum_{n=-\infty}^{+\infty} V_n^{(1)} \boldsymbol{e}_n(z) + \sum_{n \neq 0} Z_{n,s}^{(1)} I_n^{(1)} \boldsymbol{e}_n(z) = \sum_{n \neq 0} Z_{n,s}^{(1)} I_n^{(1)} \boldsymbol{e}_n(z) \tag{20}$$

$$\sum_{n=-\infty}^{+\infty} V_n^{(2)} \boldsymbol{e}_n(z) - \sum_{n \neq 0} Z_{n,s}^{(2)} I_n^{(2)} \boldsymbol{e}_n(z) = -\sum_{n \neq 0} Z_{n,s}^{(2)} I_n^{(2)} \boldsymbol{e}_n(z). \tag{21}$$

To proceed in setting up the relevant integral equation, let us now multiply (20) and (21) by $C_r^{(1)}$ and $C_r^{(2)}$, respectively, where the constant $C_r^{(m)}$ is defined as follows:

$$C_r^{(m)} = \begin{cases} \epsilon_r^{(m)} & \text{for TM modes} \\ \dfrac{1}{\mu_r^{(m)}} & \text{for TE modes.} \end{cases} \tag{22}$$

The subscript r in (22) stands for "relative," and $m = 1$ or 2 for $x \leq 0$ or $x \geq 0$, respectively. Adding the resulting expressions, we obtain

$$\sum_{n=-\infty}^{+\infty} V_n \left(C_r^{(1)} + C_r^{(2)}\right) \boldsymbol{e}_n(z) + \sum_{n \neq 0} G_n \left(I_n^{(1)} - I_n^{(2)}\right) \boldsymbol{e}_n(z) = \sum_{n \neq 0} G_n \left(I_n^{(1)} - I_n^{(2)}\right) \boldsymbol{e}_n(z) \tag{23}$$

where we have taken into account the fact that the discontinuity is purely transverse, so that

$$V_n^{(1)} = V_n^{(2)} = V_n \tag{24}$$

and the coefficient G_n is defined according to

$$G_n = C_r^{(1)} Z_{n,s}^{(1)} = C_r^{(2)} Z_{n,s}^{(2)} = \begin{cases} \dfrac{2\pi|n|}{j\omega\epsilon_0 p} & \text{for TM modes} \\ \dfrac{j\omega\mu_0 p}{2\pi|n|} & \text{for TE modes.} \end{cases} \tag{25}$$

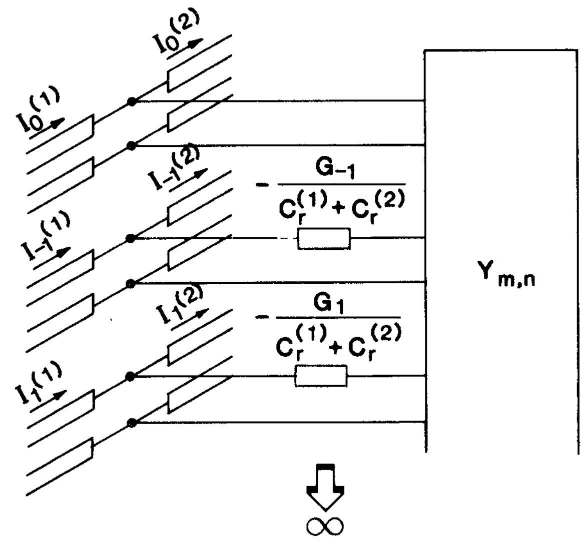

Fig. 6. An alternative approach is the obstacle formulation that leads to a network description of the scattering problem that can be viewed as the "dual" of the one shown in Fig. 5.

Finally, multiplying (23) by $1/(C_r^{(1)}+C_r^{(2)})$ we obtain the useful expression

$$\sum_{n=-\infty}^{+\infty} V_n e_n(z) + \sum_{n\neq 0} \frac{G_n}{C_r^{(1)}+C_r^{(2)}}\left(I_n^{(1)}-I_n^{(2)}\right)e_n(z) = \sum_{n\neq 0} \frac{G_n}{C_r^{(1)}+C_r^{(2)}}\left(I_n^{(1)}-I_n^{(2)}\right)e_n(z). \quad (26)$$

In a fashion similar to what we did for the aperture case, let us now redefine voltages and currents according to

$$\overline{I}_n = I_n^{(1)} - I_n^{(2)}, \qquad n = 0, \pm 1, \pm 2, \cdots \quad (27)$$

$$\overline{V}_n = \begin{cases} V_0 & \text{for } n = 0 \\ V_n + \overline{I}_n \dfrac{G_n}{C_r^{(1)}+C_r^{(2)}} & \text{for } n \neq 0 \end{cases} \quad (28)$$

and note that the above definitions lead to the network description of the discontinuity shown in Fig. 6, where the currents $\overline{I}_m$ are given by

$$\overline{I}_m = \sum_{n=-\infty}^{+\infty} \overline{V}_n Y_{m,n}. \quad (29)$$

To proceed, let us employ the well-known transform relation between the current I_m of the mth mode and a magnetic field $\boldsymbol{H}$

$$I_m = \int_D (-\boldsymbol{x}_0 \times \boldsymbol{H}) \cdot \boldsymbol{e}_m^*(z')\,dz' \quad (30)$$

where the domain D is the cross section of the unit cell, and then substitute (30) into (26), using the definitions introduced in (27) and (28). We then find

$$\sum_{n=-\infty}^{+\infty} \overline{V}_n e_n(z) = \int_{\text{obstacle}} \left[-\boldsymbol{x}_0 \times \left(\boldsymbol{H}_t^{(1)} - \boldsymbol{H}_t^{(2)}\right)\right] \cdot \sum_{m\neq 0} \frac{G_m}{C_r^{(1)}+C_r^{(2)}} \boldsymbol{e}_m(z)\boldsymbol{e}_m^*(z')\,dz' \quad (31)$$

where we have taken into account the continuity of the transverse magnetic field in the aperture. Remembering again that linearity holds, it is now appropriate to introduce the function $N_n(z')$ according to

$$-\boldsymbol{x}_0 \times \left(\boldsymbol{H}_t^{(1)} - \boldsymbol{H}_t^{(2)}\right) = \boldsymbol{B}_0(z') \sum_{n=-\infty}^{+\infty} \overline{V}_n N_n(z') \quad (32)$$

where the vector function $\boldsymbol{B}_0$ is a suitable function to be defined at a later stage. Substituting (32) into (31) and comparing like coefficients of $\overline{V}_n$, we finally deduce the following set of partial integral equations:

$$\boldsymbol{e}_n(z) = \int_{\text{obstacle}} N_n(z')\boldsymbol{B}_0(z') \cdot \sum_{n\neq 0} \frac{G_m}{C_r^{(1)}+C_r^{(2)}} \boldsymbol{e}_m(z)\boldsymbol{e}_m^*(z')\,dz' \quad (33)$$

where $n = 0, \pm 1, \pm 2, \cdots$. Note that, as with (13), (33) has a static kernel and at the same time represents a rigorous formulation of the problem under investigation.

To complete the network phrasing of the problem, let us combine (32) and (30) to determine $\overline{I}_m$ as

$$\overline{I}_m = \sum_{n=-\infty}^{+\infty} \overline{V}_n \int_{\text{obstacle}} N_n(z')\boldsymbol{B}_0(z') \cdot \boldsymbol{e}_m^*(z')\,dz'. \quad (34)$$

Finally, comparing (34) with (29) we have

$$Y_{m,n} = \int_{\text{obstacle}} N_n(z')\boldsymbol{B}_0(z') \cdot \boldsymbol{e}_m^*(z')\,dz' \quad (35)$$

where $Y_{m,n}$ is the generic element of the admittance matrix description introduced earlier in Fig. 6 together with (29). Equation (35), together with equation (33) and the network in Fig. 6, constitutes the formal solution of the obstacle formulation of the original scattering problem.

V. Explicit Expressions for TE and TM Incident Waves

Let us first summarize the general basic equations derived in the previous sections. For the *aperture formulation*, the partial integral equation that must be solved for the partial aperture magnetic current M_n is given by (13), and the expression for the impedance matrix element $Z_{m,n}$ in terms of M_n by (15). The only other elements in the equivalent network formulation shown in Fig. 5 are the static characteristic admittances that are defined in (3) and (4).

In the *obstacle formulation*, the partial integral equation for quantity N_n appears as (33), and the expression for the admittance matrix element $Y_{m,n}$ in terms of N_n is given by (35). The other elements in the equivalent network formulation shown in Fig. 6 are defined in (22) and (25).

To obtain explicit expressions for TE and TM incident polarizations, we must utilize relations (3) and (4) for the static characteristic impedances and expressions (A9) and (A10) for the vector mode functions from the Appendix and substitute them into the equations mentioned above. Four specific cases are possible, combining aperture or obstacle formulations with TM or TE excitation.

Case 1: TM Excitation and Aperture Formulation

$$e^{-j(2n\pi/p)z} = \int_{ap} M_n(z') \cdot \sum_{m \neq 0} j\frac{\omega\epsilon_0 p}{2\pi}\left(\epsilon_r^{(1)} + \epsilon_r^{(2)}\right)\frac{e^{-j(2m\pi/p)(z-z')}}{|m|} dz' \tag{36}$$

$$Z_{m,n} = \int_{ap} M_n(z') e^{j(2m\pi/p)z'} dz' \tag{37}$$

$$A_0(z') \cdot \frac{-y_0 e^{jk_{z,0}z'}}{\sqrt{p}} = 1. \tag{38}$$

Case 2: TM Excitation and Obstacle Formulation

$$e^{-j(2n\pi/p)z} = \int_{ob} N_n(z') \cdot \sum_{m \neq 0} \frac{2\pi}{j\omega\epsilon_0 p}\frac{1}{\epsilon_r^{(1)} + \epsilon_r^{(2)}}|m| e^{-j(2m\pi/p)(z-z')} dz' \tag{39}$$

$$Y_{m,n} = \int_{ob} N_n(z') e^{j(2m\pi/p)z'} dz' \tag{40}$$

$$B_0(z') \cdot \frac{z_0 e^{jk_{z,0}z'}}{\sqrt{p}} = 1. \tag{41}$$

Case 3: TE Excitation and Aperture Formulation

$$e^{-j(2n\pi/p)z} = \int_{ap} M_n(z') \sum_{m \neq 0} \frac{2\pi}{j\omega\mu_0 p}\left(\frac{1}{\mu_r^{(1)}} + \frac{1}{\mu_r^{(2)}}\right) \cdot |m| e^{-j(2m\pi/p)(z-z')} dz' \tag{42}$$

$$Z_{m,n} = \int_{ap} M_n(z') e^{j(2m\pi/p)z'} dz' \tag{43}$$

$$A_0(z') \cdot \frac{z_0 e^{jk_{z,0}z'}}{\sqrt{p}} = 1. \tag{44}$$

Case 4: TE Excitation and Obstacle Formulation

$$e^{-j(2n\pi/p)z} = \int_{ob} N_n(z') \cdot \sum_{m \neq 0} \frac{j\omega\mu_0 p}{2\pi}\frac{1}{\frac{1}{\mu_r^{(1)}} + \frac{1}{\mu_r^{(2)}}}\frac{e^{-j(2m\pi/p)(z-z')}}{|m|} dz' \tag{45}$$

$$Y_{m,n} = \int_{ob} N_n(z') e^{j(2m\pi/p)z'} dz' \tag{46}$$

$$B_0(z') \cdot \frac{y_0 e^{jk_{z,0}z'}}{\sqrt{p}} = 1. \tag{47}$$

In conclusion, the original scattering problem has been reduced to the solution of two partial integral equations of the types

$$e^{-j(2n\pi/p)z} = \int_{-b/2}^{b/2} f_n^{(1)}(z') B^{(1)} \sum_{m \neq 0} \frac{e^{-j(2m\pi/p)(z-z')}}{|m|} dz' \tag{48}$$

$$e^{-j(2n\pi/p)z} = \int_{-b/2}^{b/2} f_n^{(2)}(z') B^{(2)} \sum_{m \neq 0} |m| e^{-j(2m\pi/p)(z-z')} dz'. \tag{49}$$

In fact, we note that to obtain (36), (39), (42), and (45) from (48) and (49), one has only to use the appropriate expressions for $f_n^{(m)}(z')$, $B^{(n)}$, and b.

Note that this mathematical observation is meaningful also from a physical point of view. In the initial formulation of the problem, we started by considering two different phrasings, namely, the aperture and the obstacle phrasing. Depending on the polarization of the incident plane wave, each of these two phrasings has a different electromagnetic significance. For the obstacle phrasing, for instance, if the incident plane wave is TE, with the electric field parallel to the metallic strips, we always see the grating as an "electrically large" discontinuity. This is true even for very narrow metallic strips since they tend to short-circuit the incident electric field; in this case, therefore, even physically small metal strips correspond to an electrically large discontinuity. If the incident plane wave is TM, however, the exciting electric field is perpendicular to the metal strips, so that the grating of narrow strips hardly sees the electric field and the effective discontinuity becomes "electrically small." Even for wider strips, the reflected power produced by the strips for TM wave incidence will be less than that expected on geometric grounds alone, and certainly less than that for TE incidence. It is therefore appropriate to characterize an obstacle discontinuity as "electrically large" for TE incidence and "electrically small" for TM incidence. For the other phrasing (the aperture phrasing) the reverse is true: a TE excitation sees the grating as an electrically small discontinuity, while a TM excitation sees the grating as an electrically large discontinuity. For TE excitation, therefore, an aperture that is small in size is at the same time physically small and electrically small, while for TM excitation an aperture that is physically small is seen as an electrically large discontinuity. Of the two integral equations above, therefore, (48) corresponds physically to the "electrically large" case and (49) to the "electrically small" case.

Both (48) and (49) are Fredholm integral equations of the first kind with singular kernels. In Part II of this paper, we present two approximate solution procedures, one for each equation, that lead to very simple and accurate network representations for the multimode grating on an air–dielectric interface.

Appendix
The Vector Mode Functions

As a consequence of the unit cell approach, the phase difference between opposite walls on the equivalent waveguide that represents a unit cell is the same for either TE or TM excitation since it depends on the angle of incidence. Therefore, apart from a change of direction, the vector mode functions have the same form for both polarizations. As a result, we outline here only the derivation for the TM case. For a more detailed mathematical derivation of this case, the reader is referred to [19].

It is well known that for any cross-section-invariant structure under TM excitation, the vector mode functions $\boldsymbol{e}_n$ and $\boldsymbol{h}_n$ can be derived from the scalar mode function $\phi_n(z)$ via the expressions

$$\boldsymbol{e}_n(z) = \frac{-\nabla_t \phi_n(z)}{k_{z,n}} \tag{A1}$$

$$\boldsymbol{h}_n(z) = \boldsymbol{x}_0 \times \boldsymbol{e}_n(z) \tag{A2}$$

where $k_{z,n}$ is the wavenumber in the transverse direction (z direction in this case; there is no variation in the y direction). It can be shown that the scalar mode function $\phi_n(z)$ must satisfy the equation

$$\left(\nabla_t^2 + k_{z,n}^2\right)\phi_n(z) = 0 \tag{A3}$$

together with the proper boundary conditions. Due to the presence of the grating, the solution of (A3) can be written in the form

$$\phi_n(z) = e^{-jk_{z,0}z} f_n(z) \tag{A4}$$

where

$$k_{z,0} = k_0 \sin\theta \tag{A5}$$

θ being the angle of incidence measured from the normal and

$$f_n(z+p) = f_n(z) \tag{A6}$$

where p is the grating period. Introducing (A4), (A5), and (A6) into (A3), we obtain

$$\phi_n(z) = A e^{-jk_{z,0}z} e^{-j(2n\pi/p)z} \tag{A7}$$

where, to evaluate the constant A, we employ the normalization condition

$$\int_{-p/2}^{p/2} \phi_n(z')\phi_n^*(z')\, dz' = 1. \tag{A8}$$

Finally, for the vector mode functions, we obtain the expressions

$$\boldsymbol{e}_n(z) = z_0 \frac{e^{-jk_{z,n}z}}{\sqrt{p}} \tag{A9}$$

$$\boldsymbol{h}_n(z) = -y_0 \frac{e^{-jk_{z,n}z}}{\sqrt{p}} \tag{A10}$$

where the wavenumber $k_{z,n}$ is given by

$$k_{z,n} = k_0 \sin\theta + 2n\pi/p. \tag{A11}$$

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